Rotational Variable Mass System (RVMS):

A novel intro of a new theory (VSA-PSR Theory) with a concept of moment of mass and its applications - a first time feasibility study

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Abstract

In this article, Rotational Variable Mass System (RVMS): a novel theory (VSA-PSR Theory) was introduced with an extrapolation of a concept: Moment of mass (ϖ). Also, the important applications and active part of moment of mass in various physical parameters were introduced as a first-time feasibility study.

Introduction

Moment of mass (ϖ) is the active part of the moment of inertia, i.e., moment of inertia is the moment of moment of mass.

i.e., moment of mass $\varpi = \int rdm$ (or) $\sum_{i=1}^{n} m_i r_i$

Simply, $\varpi = mr$..(1)

Hereafter, we may call ϖ as "agemo".

Since it looks like a reciprocal of ω (Omega), moment of mass is the basic concept of rotational dynamics (or) mechanics, even it can be the basic for planetary motions as well as satellites.

Basic derivation of moment of mass from fundamental theory

We knew that, centripetal (C.P) force and centrifugal (C.F) force both must be equalize for the perfect circular motion of a rigid body of mass m with radius "r", now we may write

$$C.P\ (or) C.F = \frac{mv^2}{r} = m\frac{r^2\omega^2}{r}$$

(Since $v = r\omega$)

$C.P\ (or)\ C.F = mr\omega^2 = \varpi\omega^2$ .... (2)

Rotational Variable Mass System (RVMS)

A rotational rigid body simultaneously there is a change in mass, either increment (or) decrement. The best example for this system is Deepavali festival fire cracker named "Ground chakra", it is a variable mass and variable radius system. Let us consider an acceleration of a rotational rigid body with an assumption that the velocity of the ejected gas by the burning of the fuel is $\vec{u}$ (as initial velocity). According to Newton's third law of force decreases in the mass of the fuel increases the angular velocity (ω) of the ground chakra. So, the ground chakra rotates very fast, when the time progresses. In this case, we can write as

$$\tau = \omega a = \omega \frac{dv}{dt} \quad \text{..} (3)$$

Also, $\tau = rF = rma = \omega \mathrm{a}$

(Since $I = mr^2$)

When the fuel content decreases, ultimately the angular velocity (ω) of the ground chakra also increases and reaches an ultimate level ($\vec{v}$ as final velocity). In this case, the equation (3) can be written as,

$$\omega \frac{dv}{dt} = (u - v)\frac{d\omega}{dt} = -u' \frac{d\omega}{dt}$$

This torque equation is preferentially with respect to mass decrement. Where $u'$is the net velocity of the ejected mass relative to the ground chakra, (Here $v = u + u'; u - v = -u'$ by classical mechanics). The (-ve) sign indicates the both velocities are opposite in directions, (Also, v >> u). From equation (3), we can write,

$$\frac{d\omega}{\omega} = -\frac{dv}{u'}$$

Integrating on both sides, we get

$$\int_{\varpi_0}^{\varpi} \frac{d\varpi}{\varpi} = -\int_{u}^{v} \frac{dv}{u'}$$

Also, ϖ_0 is the initial moment of mass, at time(t) = 0, i.e., at starting stage.

ϖ is the any instantaneous moment of mass, at any time 't'.

Where, ϖ_0, u are initial lower limit values, ϖ, v are final limit values in this integration operation.

$$[log\,\varpi\,]_{\varpi_0}^{\varpi} = -\frac{1}{u'}\int_{u}^{v} dv$$

$$log\,\varpi - log\,\varpi_0 = -\frac{1}{u'}[v]_u^v$$

$$\left[log\,\frac{\varpi}{\varpi_0}\right] = -\frac{1}{u'}[v - u] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4)}$$

$$u'\left[log\,\frac{\varpi}{\varpi_0}\right] = -(v - u)$$

From equation (4), we can write,

$$\therefore\ \frac{-(v-u)}{u'} = log\,\frac{\varpi}{\varpi_0}$$

by mathematical simplification, we can easily get equation 4a

$$\therefore\ v = u - u'\left[log\,\frac{\varpi}{\varpi_0}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4a)}$$

This equation 4a is used to calculate the final velocity of RVMS.

We can write Equation 4a in terms of angular velocity “ω"

$$V = u - u^{'} \left[\log \text{—}\right] - 4a$$

$$V = r\omega$$

$$u = r\omega_0$$

$$u' = r\omega'$$

$$r\omega = r\omega_0 - r\omega^{'} \left[\log \frac{\omega}{\omega_0}\right]$$

$$r\omega = r(\omega_0 - \ \omega^{'} \ \left[\log \frac{\omega}{\omega_0}\right])$$

$$\omega = \ \omega_0 \ - \ \ \omega^{'} \left[log \frac{\omega}{\omega_0}\right] \cdots\cdots\cdots\cdots\cdots 4b$$

Also, We can

$$\omega - \ \omega_0 \ = \ - \omega^{'} \ [\log \frac{\omega}{\omega_0}]$$

$$-\left[\frac{\omega - \omega_0}{\omega^{'}}\right] = \ log_e \frac{\omega}{\omega_0}$$

$$\frac{\omega}{\omega_0} = \ e^{-\left[\frac{\omega - \omega_0}{\omega^{'}}\right]}$$

$$\omega \ = \ \omega_{\,0} \, e^{-\left[\frac{\omega - \omega_0}{\omega^{'}}\right]} \ \cdots\cdots\cdots\cdots\cdots 4c$$

Also we can write as,

$$\varpi = \varpi_0 e^{-\left(\frac{v-u}{u'}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

This equation implies that the change in velocity of the ground chakra, when its moment of mass changes from ϖ_0 to ϖ, the velocity of it changes from $\vec{u}$ to $\vec{v}$.

i.e., moment of masses changes from ϖ_0 to ϖ.

Applications

We may write the linear and rotational physical parameters in terms of moment of mass,

(i) Torque (τ)as

$$\tau = I \propto = r \times F = rF = rma = mra$$

$$\therefore \quad \tau = \varpi a$$

(ii) Angular momentum (L) as

$$L = r \times P = rP = rmV = \varpi V$$

Therefore, $L = \varpi V$

(iii) Rotational kinetic energy (K.E) as

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}mr^2\omega^2$$

$$K.E = \frac{1}{2}r\,\varpi\omega^2$$

Conclusion

As a result of 30 years of research and based on the literature survey, also to the best of our knowledge, we may stated that this is the first time that a novel theory (VSA-PSR Theory) about Rotational Variable Mass System (RVMS) was proposed.

Reference

[1] Fundamentals of Physics by David Halliday, R. Resuick and J. Walker, 6th Edition, Wiley, New York, 2001.